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PROGRAM LEVEL OF AGROCENOSIS MANAGEMENT, TAKING INTO ACCOUNT THE IMPACT OF WEEDS ON CROPS

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Abstract

In modern crop production, a traditional paradigm of separate management of crops and weeds as part of single agroecosis dominates. However, mineral fertilizers simultaneously stimulates the growth and development of crops and weeds, and herbicides suppress the growth of both cultivated plants and weeds. This leads to significant yield losses and waste of fertilizers and herbicides. The purpose of this study is to develop a theoretical basis for solving the problem of managing agroecoses, which include the main crop and weeds. The solution of this problem is aimed at eliminating the limitations of the existing paradigm of separate management of crops and weeds in the agroecosis. Previously, we have developed a theory of management of agro-technologies, in which the object of management is an agricultural crop without considering the role of weeds in the agroecoses. In accordance with this theory, the crop management is carried out at strategic, program and real-time levels. In the presented work, for the first time, the problem of managing agroecoses at the program level during one growing season is posed and solved. The essence of the approach lies in the development of the programs, which are sequences of technological operations for the application of mineral fertilizers, irrigation and herbicide treatments, providing a given crop yield with minimal expenditure of resources. To solve this problem, in the previously developed theory, mathematical models for crop parameters are modified to reflect the effect of herbicides. In addition, a model of the parameters of dominant weed species was introduced into the control task, in which, in addition to the doses of herbicide treatments, the effect of the doses of mineral fertilizers is reflected. The mathematical model for the soil environment, which takes into account the influence of the parameters of the state of the cultivated crop and weeds, has also undergone significant refinement. The problem is solved on the example of sowing spring wheat as part of an agroecoses. The presence of several spring wheat phenological phases needs to transform the structure and parameters of the mathematical models used for all phenophases. This, in turn, needs to solve the problem of forming optimal programs for managing agroecoses separately for each interphase period and combine the received private programs into a single program. As a method for solving the problem, the Pontryagin's maximum principle is used in combination with a dynamic programming scheme (from the end of the growing season to its beginning). The structural complexity of the control object, which is an agricultural field with agroecoses, necessitates solving the problem of program control in three stages. At the first stage, a program is formed to change the parameters of the soil environment, which ensures the achievement of the required crop yield. At this stage, the effect of herbicide treatments on the state of crop sowing is not considered. At the second stage, a sequence of technological operations is found that provides the best approximation of soil parameters to the optimal program obtained at the first stage. Finally, at the third stage, the optimal sequence of herbicide treatments performed simultaneously with other technological operations is found. To consider the influence of these processing, the programs obtained at the first two stages are refined until the convergence of the solution of the entire problem is obtained.

Keywords: program control, agroecoses, mineral fertilizers, herbicides, mathematical models, control algorithms

In modern crop production, the traditional paradigm of separate management of crops and weeds as part of one agroecosis has long been established. The development of precision farming (PF) stimulates creation of an effective theory

for agricultural technology management. But this mainly concerns the crops managing. Many provisions of modern theory have already been developed, from a general concept of control to algorithms for control at various time levels [1, 2]. According to this theory, it becomes possible to form control programs which are sequences of technological operations with the optimal level of fertilizer doses and irrigation rates. As for the management of weeds, the progress in the development of the theory is more modest, and to date, the optimal doses of herbicide treatments have not yet been scientifically substantiated. Note, the existing theoretical base for managing the state of agricultural crops does not take into account the fact that weeds are present in the agrocenosis, and the application of fertilizers and irrigation stimulate weed growth and development along with the crop. And vice versa, the treatment with herbicides inhibits not only weeds but also the main crop of agrocenosis. As a result, such separate management leads to significant crop losses, overspending of fertilizers and herbicides, and deterioration of environmental performance. The recent appearance of a sufficient number of publications on the joint application of fertilizers and herbicides shows that technological science is striving to eliminate the shortcomings of the existing separate management paradigm [3-5]. This poses new challenges for the science of managing agricultural technologies, forcing to consider a field with an agrocenosis as a single management object (MO).

According to the proposed concept of PF management, the overall task includes four levels of sub-tasks solved at different time scales [1]. At the top 1st level, crop rotation managing on an annual scale must be solved; at the 2nd level, implemented on a daily scale at one vegetation interval, the program control is solved. The tasks of two planning levels are decided in advance, out of real time. Tasks of the 3rd and 4th levels, where technological operations are directly formed, are implemented in a real time mode. Of all the above levels, the program level of control is the key one, since it is through it that the strategic tasks of management are connected with real-time tasks. At the program level of management, an optimal sequence of technological operations is planned to ensure the achievement of the desired result [2].

In accordance with the proposed concept of management at the program level, a field with an agricultural crop is an MO. However, the concept does not take into account the fact that in the same field as part of the agrocenosis, in addition to the main crop, annual and perennial weeds grow. They compete with the plants of the cultivated crop for moisture and nutrients, and crop losses from weed infestation can exceed 50%. Therefore, the optimal technological programs of operations in the considered vegetation interval should include not only fertilization and irrigation operations, but also herbicide treatments. Such programs should be formed given the fact that mineral nutrition stimulates the growth of both cultivated plants and weeds, and herbicides not only suppress the growth and development of weeds, but also act depressingly on cultivated plants.

The formation of a unified program of simultaneous application of mineral fertilizers and herbicide treatments, coordinated according to the state of the main crop and weeds, will avoid crop losses and cost overruns of mineral fertilizers and herbicides. In addition, optimization of fertilizer doses that meet the biological needs of the crop in nutrients activates metabolic processes, accelerates the inactivation of the incoming herbicide and increases the resistance of the protected plant to it. The protected crop, due to more intensive accumulation of organic mass,

receives a significantly lower dose of herbicide per unit mass, that is, there is a decrease in the herbicide content in tissues during growth, and smaller amounts of the drug, given optimal metabolism, are inactivated faster. Optimal nutritional conditions also increase the overall biological competitiveness of the crop against weeds [3-5].

An analysis of foreign publications has shown that today only particular aspects and little interconnected tasks of managing the crop state have been developed, including models for estimating and predicting crop biomass [7, 9], principles of zonal management of nitrogen nutrition and risks [10-12, 18], general principles of agricultural production resource management with resource models [13-17, 19]. Thereof, it can be argued that a unified theoretical framework for managing agricultural technologies in PF has not yet been developed.

This is due to insufficient knowledge of agrocenosis as a single MO for which complex models of the relationship between the state of crops and weeds in sowing have not yet been created. A large review paper [20] discusses the nature and practice of using a full range of simulation models for ecology, biology and weed control, and the use of such models for information and decision support. As a rule, plant protection specialists proceed from a quantitative assessment of the population density and aggressiveness of the weeds. An important place is occupied by models that predict crop losses and thresholds for determining the methods and timing of weed control measures [21, 22]. Several approaches have been implemented based on the relationship between weed density in crops and yield loss, and it has been proven that this relationship is described by a rectangular hyperbola [23].

Weed biomass can be a reliable predictor of crop loss [24, 25]. The higher it is (regardless of the density of weeds in the agrocenosis), the more the crop yield decreases. However, accounting for weed biomass is time consuming and difficult to control in the field. Another problem in predicting crop losses from weed biomass is that there is no clear understanding of how much of this biomass should be considered. An example of predicting the impact of weeds and crop losses is the model proposed by M.J. Krop and J.T. Spitters [26] for sugar beet.

Adaptive changes in weeds are another factor that must be taken into account when analyzing the weed-crop system. With an increase in the competitiveness of weeds, the potential of agricultural crops will decrease. As a consequence, the accuracy of predictive crop loss models due to weeds will gradually decrease [27, 28]. Therefore, to correctly take into account the mutual influence of weeds and crops, periodic parameterization and recalibration of models is necessary.

In practice, solving the problem of crop losses from weeds requires knowledge of the species-specific, time-varying relationships between weeds and cultivated plant species, understanding the short-term and long-term consequences of the adopted tactics of protective measures [29, 30]. Without similarly interpreted multifactorial events, it is impossible to design protective programs that are effective, functional, and will not harm non-target organisms [31-34].

The need for decision support tools (DST) for practitioners is especially great. Such DST should combine models of weed population dynamics, economic efficiency of technology [35-37] and its impact on the environment [36, 37]. Such DST will allow practitioners to model new management options in local conditions, adapt sustainable management concepts to the characteristics of resident weeds [38-40], and compare the likely short-term outcomes of possible interventions. Some

DSTs model the short-term results of mechanical control tactics [31, 41-43]. DST tactics are most effective for chemical weed control [36, 37, 42, 44, 45]. These DSTs provide guidance to choose a number of aspects of herbicide application [31, 38, 46-49]. In addition, many solutions offer projections on financial interventions [31, 42, 46-49].

Decision support tools for preventive management serve two main purposes: first, they predict weed infestation and returns from different interventions over several years; second, they provide these forecasts within the scope of the respective intended target [40-44].

The analysis shows that, with all the breadth of covering the problem, the proposed tools do not include reasonable mathematical models and the choice of achievable management goals, optimality criteria and effective control algorithms to develop of technological operation programs that ensure management goals.

In the proposed work, for the first time, we pose and solve the problem of unified management of agrocenosis at the program level for one growing season. To solve it, in the previously developed theory of management of agricultural technologies, the mathematical models of the crop state are modified to reflect the effect of herbicides. In addition, a model for the parameters of the dominant species of weeds was used, in which, together with the dosage of herbicides, the influence of mineral fertilizers is considered. We have proposed a novel three-stage procedure for the program management of agrocenosis. At the first stage, a program is formed to change the soil conditions, ensuring the required crop yield. At this stage, the effect of herbicides on the crop sowing is not taken into account. At the second stage, a sequence of technological operations is found that provides the best approximation of soil parameters to the optimal program obtained at the first stage. Finally, at the third stage, the optimal sequence of herbicide treatments performed simultaneously with other technological operations is found.

The purpose of this work was to further develop the theory of programmed management for an agrocenosis with spring wheat crops as the MO to be controlled.

Materials and methods. The classical control theory with dynamic programming and the Pontryagin maximum principle [6] were used. According to this theory, the starting point for solving any control problem is the choice of an achievable goal. When we are dealing with agricultural technology, such a goal can only be to obtain a given crop yield at the end of the growing season. Any control task is based on the mathematical description of the MO. In the case under consideration, this is an agricultural field with an agrocenosis which includes a crop of spring wheat. The fundamental basis for solving problems of program control are mathematical models that describe the dynamics of the parameters of the state of the control object (MO). In addition to the mathematical model of the main crop, the MO should be supplemented by a model of weeds. Such models should reflect the influence of external uncontrolled disturbances, the influence of controlled factors on the MO parameters and take into account their interconnection through the soil environment.

To achieve the above management goal, it is necessary to select the most important goal-forming parameters of the crop sowing. The crop under consideration is characterized both by continual state parameters, which include the crop biomass and soil parameters, and by structural state parameters, including phenophases. For spring wheat, depending on the duration of the interval on a daily time scale t , these are at $t \in (0-7)$ $s = 1$, sowing; at $t \in (11-13)$ $s = 2$, seedlings (1st, 2nd, 3rd leaves); at $t \in (21-29)$ $s = 3$, tillering; at $t = 30$ $s = 4$, stem extension; at $t \in (31-32)$ $s = 5$, internode; $t = 37$ $s = 6$, flag leaf; at $t = 39$ $s = 7$, ligula; at

$t = 49$ s = 8, leaf sinus opening; at $t \in (51-59)$ s = 9, heading; at $t \in (61-69)$ s = 10, flowering; at $t \in (71-75)$ s = 11, milky ripeness; at $t \in (85-86)$ s = 12, wax ripeness; at $t \geq 86$ s = 13, full ripeness.

The entire growing season, depending on the structure of the sowing biomass, can be divided into two time intervals, from the 2nd to the 9th and from the 9th to the 13th phenophase. Fertilization, herbicide treatments and watering are carried out at fixed times of the onset of the following pre-selected phenological phases: s = 3 (tillering), s = 9 (earring), s = 10 (flowering), s = 11 (milky ripeness). This makes it necessary to break down the entire management interval into four intervals between the selected phenophases: 1st from tillering to heading (T_3, T_9), 2nd from heading to flowering (T_9, T_{10}), 3rd from flowering to milky ripeness (T_{10}, T_{11}), and 4th from milky ripeness to full ripeness (T_{11}, T_{13}).

For the first time interval (the phenophases from the 3rd to the 9th), the model for dynamics of crop biomass structure parameters has the form [1, 2]:

$$\begin{aligned} \begin{bmatrix} \dot{x}_{1m} \\ \dot{x}_{2m} \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_m \begin{bmatrix} x(t)_{1m} \\ x(t)_{2m} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix}_m \begin{bmatrix} v_N(t) \\ v_K(t) \\ v_P(t) \\ v_{Mg}(t) \\ v_s(t) \end{bmatrix} + \\ &+ \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}_m \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} - \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}_m \begin{bmatrix} g_1(t) \\ g_2(t) \end{bmatrix}_m, \quad t \in (T_3, T_9), \end{aligned} \quad (1)$$

where x_{1m} is the average planting biomass density (yield), $t \cdot \text{ha}^{-1}$ over the area of the field; x_{2m} is the density of the wet mass of crops averaged over the area of the field, $c \cdot \text{ha}^{-1}$; external disturbances in both blocks are f_1 , the average daily air temperature, $^{\circ}\text{C}$; f_2 , the average daily radiation level, $\text{W} \cdot (\text{m}^2 \cdot \text{h})^{-1}$; f_3 , the average daily precipitation intensity, mm. Chemical parameters of the soil are v_N , the nitrogen content, $\text{kg} \cdot \text{ha}^{-1}$; v_K , the potassium content, $\text{kg} \cdot \text{ha}^{-1}$; v_P , the phosphorus content, $\text{kg} \cdot \text{ha}^{-1}$; v_{Mg} , the magnesium content, $\text{kg} \cdot \text{ha}^{-1}$; v_s is the moisture reserve in the soil, mm; $g_{1m}(t)$, $g_{2m}(t)$ are doses of a herbicide, $\text{g} \cdot \text{ha}^{-1}$. Because of research and methodological reasons of the work, we do not disclose the types of herbicides here, since the approach we develop is applicable to any herbicides.

For further use, it is convenient to represent the model (1) in the canonical symbolic vector-matrix form, where all variables are combined into vectors, and parameters into the corresponding matrices:

$$\dot{X}_m = A_m X_m(t) + B_m V(t) + C_m F(t) - D_m G_m(t). \quad (2)$$

For intervals from the second, the (T_9, T_{10}), (T_{10}, T_{11}) and (T_{11}, T_{13}) the models of the dynamics of crop biomass structure parameters have a general form and differ only in the values of the parameters [1, 2]:

$$\begin{aligned} \begin{bmatrix} \dot{x}_{1u} \\ \dot{x}_{2u} \\ \dot{x}_{3u} \end{bmatrix}_j &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_j \begin{bmatrix} x(t)_{1u} \\ x(t)_{2u} \\ x(t)_{3u} \end{bmatrix}_j + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \end{bmatrix}_j \begin{bmatrix} v_N(t) \\ v_K(t) \\ v_P(t) \\ v_{Mg}(t) \\ v_s(t) \end{bmatrix}_j + \\ &+ \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}_j \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}_j - \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \end{bmatrix}_j \begin{bmatrix} g_1(t) \\ g_2(t) \end{bmatrix}_j, \\ &j = 1, t \in (T_9, T_{10}); j = 2, t \in (T_{10}, T_{11}), j = 3, t \in (T_{11}, T_{13}). \end{aligned} \quad [3]$$

In this model, the x_{1u} is the average density of the crop biomass over the area of the field, $c \cdot \text{ha}^{-1}$; x_{2u} is the density of the wet mass of crops averaged over the area of the field, $c \cdot \text{ha}^{-1}$; x_{3u} is the density of the mass of ears (yield) averaged over the area of the field, $c \cdot \text{ha}^{-1}$; external disturbances in both blocks are f_1 - average daily air temperature, $^{\circ}\text{C}$; f_2 - average daily radiation level, $\text{W} \cdot (\text{m}^2 \cdot \text{h})^{-1}$; f_3 is the average daily precipitation intensity, mm ; chemical parameters of the soil are : v_N - nitrogen content in the soil, $\text{kg} \cdot \text{ha}^{-1}$; v_K is the content of potassium in the soil, $\text{kg} \cdot \text{ha}^{-1}$; v_P is the content of phosphorus in the soil, $\text{kg} \cdot \text{ha}^{-1}$; v_{Mg} is the content of magnesium in the soil, $\text{kg} \cdot \text{ha}^{-1}$; v_5 is the moisture content in the soil, mm ; $g_{1u}(t)$, $g_{2u}(t)$ are doses of herbicide treatment, $\text{g} \cdot \text{ha}^{-1}$; $j = 1, 2, 3$ are the numbers of control intervals after the heading phase.

The canonical symbolic vector-matrix form of the model has the following form [3]:

$$\dot{X}_{ij} = A_{ij} X_{ij}(t) + B_{ij} V_{ij}(t) + C_{ij} F(t) - D_{ij} G_{ij}(t). \quad (4)$$

As mentioned above, in addition to the model (4), to solve the problem, a dynamic model of the biomass of the dominant weed species is required, the vector-matrix form of which has the form

$$\dot{S}_j = A_{sj} S_j(T) + B_s V_j(T) - B_{gj} G_j(t) + C_{sj} F(t), \quad (5)$$

Where $S^T = [s_1 \ s_2]$ is the biomass vector of the dominant weed species, T is the transposition index of the vector or matrix,

$$A_s = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}_s, B_s = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix}_s, B_g = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}_g, C_s = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{bmatrix}_s$$

are matrices of model parameters.

The model (5) includes the states of two dominant weed species. For other conditions, their number and types may be different, which only refines the structure of the algorithms, but does not change the general approach to solving the problem.

Models (2), (4) and (5) represent the main block of MO state parameters. In addition to this block, the MO contains a block of control transfer, which is the soil environment (SE). It is through this block that crop plants and weeds compete for nutrients and moisture.

The model of dynamics of soil state parameters for phenophases 3 to 9 takes the following form [1]:

$$\begin{bmatrix} \dot{v}_N \\ \dot{v}_K \\ \dot{v}_P \\ \dot{v}_{Mg} \\ \dot{v}_5 \end{bmatrix}_{3,9} = \begin{bmatrix} a_{11} & 0 & 0 & 0 & a_{15} \\ 0 & a_{22} & 0 & 0 & a_{25} \\ 0 & 0 & a_{33} & 0 & a_{35} \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}_{3,9} \begin{bmatrix} v_N \\ v_K \\ v_P \\ v_{Mg} \\ v_5 \end{bmatrix}_{3,9} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{3,9} \begin{bmatrix} d_N(t) \\ d_K(t) \\ d_P(t) \\ d_{Mg}(t) \\ d_W(t) \end{bmatrix}_{3,9} + \quad (6)$$

$$+ \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ 0 & 0 & c_{33} \\ 0 & 0 & c_{43} \\ c_{51} & c_{52} & 1 \end{bmatrix}_{3,9} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} - \begin{bmatrix} m_{11} & 0 \\ m_{21} & 0 \\ m_{31} & 0 \\ m_{41} & 0 \\ m_{51} & m_{52} \end{bmatrix}_{3,9} \begin{bmatrix} x_{1m}(t) \\ x_{2m}(t) \end{bmatrix} - \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \\ p_{31} & p_{32} \\ p_{41} & p_{42} \\ p_{51} & p_{52} \end{bmatrix}_{3,9} S(t),$$

or

$$\dot{V}_{3,9} = A_{3,9} V(t) + B_{3,9} D(T_3, T_9) + C_{3,9} F(t) - M_{3,9} X_m(t) - P_{3,9} S(t). \quad (7)$$

Models of the dynamics of soil state parameters for the intervals (T_9, T_{10}) , (T_{10}, T_{11}) and (T_{11}, T_{13}) have the same form, differing only in the values of the parameters:

$$\begin{aligned} \begin{bmatrix} \dot{v}_N \\ \dot{v}_K \\ \dot{v}_P \\ \dot{v}_{Mg} \\ \dot{v}_5 \end{bmatrix}_j &= \begin{bmatrix} a_{11} & 0 & 0 & 0 & a_{15} \\ 0 & a_{22} & 0 & 0 & a_{25} \\ 0 & 0 & a_{33} & 0 & a_{35} \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}_j \begin{bmatrix} v_N \\ v_K \\ v_P \\ v_{Mg} \\ v_5 \end{bmatrix}_j + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_j \begin{bmatrix} d_N(t) \\ d_K(t) \\ d_P(t) \\ d_{Mg}(t) \\ d_w(t) \end{bmatrix}_j + \\ &+ \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ 0 & 0 & c_{33} \\ 0 & 0 & c_{43} \\ c_{51} & c_{52} & 1 \end{bmatrix}_j \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}_j - \begin{bmatrix} m_{11} & 0 & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & 0 & m_{33} \\ m_{41} & 0 & m_{43} \\ m_{51} & m_{52} & 0 \end{bmatrix}_j \begin{bmatrix} x_{1u}(t) \\ x_{2u}(t) \\ x_{3u}(t) \end{bmatrix}_j - \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \\ p_{31} & p_{32} \\ p_{41} & p_{42} \\ p_{51} & p_{52} \end{bmatrix}_j S(t), \end{aligned} \quad (8)$$

where $dP(t)$, $dK(t)$, $dN(t)$, $dW(t)$ are the doses of nutrients (phosphorus P, potassium K, ameliorant Ca, nitrogen N and magnesium Mg, $\text{kg} \cdot \text{ha}^{-1}$, respectively) and irrigation rate, mm; $a_{11}-a_{33}$, b_2-b_3 , c_1-c_3 are model parameters estimated from experimental data; t is days.

Model (8) in compact symbolic vector-matrix form is

$$\dot{V}_j = A_j V_j(t) + B_j D_j(t) + C_j F(t) - M_j X_j(t) - P_j S(t). \quad (9)$$

The complex multidimensional structure of the MO, including models (2), (4), (5) of the main block of state parameters and blocks of control transfer (7), (9), needs to solve the program control problem in two stages [1, 2]. At the first stage, there is a program for the potential development of crop sowing throughout the growing season, which ensures the expected management goal. In this case, the influence of weeds is not taken into account, and the parameters of the state of the SE are considered as control variables without taking into account technological limitations. The control program obtained in this way serves as a guide to form a sequence of technological operations, including fertilization, watering and herbicide treatments.

Therefore, at the second stage, a sequence of technological operations is found, which should ensure the minimum deviation of the SE parameters from the optimal program obtained at the first stage. Such a decomposition of the program control greatly simplifies the synthesis of optimal control programs. In addition, the optimization results obtained at the first stage are of independent interest as a characteristic of the potential level of crop yield.

In accordance with the dynamic programming scheme [6], the task of the first stage is solved from the end of the growing season to its beginning. In this case, the goal of management is to obtain a given crop yield at the end of the growing season under the following conditions: achieving the required structure of the entire biological crop (namely, the required quantitative ratio between grain and straw), the required grain moisture content, as well as reducing the biomass of weeds in the agrocenosis to the specified level.

In the indicated state parameters, the control goal for the specified vegetation interval formally looks like this:

$$x_{1u}(T_{13}) \geq 2,1U^*, \quad x_{2u}(T_{13}) \leq 0,15U^*, \quad x_{3u}(T_{13}) \geq U^*, \quad S_{ij}(T_{13}) \leq S_{ij}^*,$$

where $U^*(T_{13})$ is given yield, $\text{c} \cdot \text{ha}^{-1}$, S_{ij}^* is given weed biomass.

The optimality criterion for the non-vegetation period from the 9th to the

13th phenophase, which meets the goal, has the following form

$$\begin{aligned} J_{uj}(T_{13}) = & [X_{uj}(T_{13}) - X_{uj}^*(T_{13})]^T G_u [X_{uj}(T_{13}) - X_{uj}^*(T_{13})] + \\ & + [S_{uj}(T_{13}) - S_{uj}^*(T_{13})]^T Q [S_{uj}(T_{13}) - S_{uj}^*(T_{13})], \end{aligned} \quad (10)$$

where $X^{*T} = [2, 1U^* \ 0, 15U^* \ U^*]$ is a vector which components are total biomass, fresh weight, grain weight (yield);

$$G_u = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \text{ is weight matrix of mass and quality components of the criterion,}$$

$$Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \text{ is weight matrix of weed biomass components,}$$

$j = 0, 1, 2, 3$ – control interval indices.

To form optimal programs for all control intervals, the Pontryagin's maximum principle [6] is used. In accordance with this method, the Hamiltonian of the system, which includes the models (4), (5) and the criterion (10), has the following form:

$$\begin{aligned} H_{uj}(t) = & \Psi_{1,uj}^T [A_{uj} X_{uj}(t) + B_{uj} V_{uj}(t) + C_{uj} F(t) - D_{uj} G_{uj}(t)] + \\ & + \Psi_{2,uj}^T [A_s S_{uj}(T) + B_s V_{uj}(T) - B_g G_j(t) + C_s F(T)]. \end{aligned} \quad (11)$$

and models of conjugate variables are

$$\dot{\Psi}_{1,uj=3,i} = -\frac{\partial H_{uj=3,i}}{\partial X_{1,uj=3,i}} = -A_{uj=3,i}^T \Psi_{1,uj=3,i}, \quad t \in (T_{11}, T_{13}), \quad \Psi_{1,uj=3,i}(T_{13}) = [X_{uj}(T_{13}) - X_{uj}^*], \quad (12)$$

$$\dot{\Psi}_{2,uj=3,i} = -\frac{\partial H_{uj=3,i}}{\partial S_{1,uj=3,i}} = -A_{sj=3,i}^T \Psi_{2,uj=3,i}, \quad t \in (T_{11}, T_{13}), \quad \Psi_{2,uj=3,i}(T_{13}) = 2[S_{uj}(T_{13}) - S_{uj}^*]. \quad (13)$$

The algorithm for the formation of an optimal program for changing the parameters of the soil environment, which ensures the management goal, includes the following iterative procedures:

1) formation of optimal programs for changing the parameters of the substation

$$\begin{aligned} V_{uj,i+1}^*(t) = & V_{uj,i}^*(t) - \Delta_i GR_{uj,i}(t), \\ GR_{uj,i}(t) = & \frac{\partial H}{\partial V_{uj,i}}(t) = B_{uj}^T \Psi_{1,uj,i}(t) + B_s^T \Psi_{2,uj,i}(t). \end{aligned} \quad (14)$$

2) search for initial conditions at the boundaries of phenophases:

$$X_{uj,i+1}^*(T_{11}) = X_{uj,i}^*(T_{11}) - \Delta_i \Psi_{1,uj,i}(T_{11}), \quad (15)$$

$$S_{uj,i+1}^*(T_{11}) = S_{uj,i}^*(T_{11}) - \Delta_i \Psi_{2,uj,i}(T_{11}). \quad (16)$$

As a result of solving the task of the first stage, an optimal program for changing the vector of SE parameters $V^*(t)$ is formed, which consists of separate pieces at four control intervals between the selected phenophases. This program corresponds to the program for changing the vector of sowing state parameters $X^*(t)$ and the program for changing the vector of weed biomass parameters $S^*(t)$ at these control intervals.

The achievable management goal at the second stage of the general solution is to ensure the closest approximation to the optimal programs for the content of nutrients and moisture in the soil obtained at the first stage of the forecast by independently choosing the doses of fertilizers and watering. Fertilization and irrigation are carried out at fixed times of the onset of the following phenological phases: $s = 3$ (tillering), $s = 9$ (heading), $s = 10$ (flowering), $s = 11$ (milky ripeness).

As for the first stage, the problem is solved separately for each control

interval, but in the forward direction - from the beginning of the growing season to its end.

Particular optimality criteria for each j -th control interval have the same form:

$$J_j = \int_{T_{1j}}^{T_{2j}} [(V_j^*(t) - V_j(t))^T G_j (V_j^*(t) - V_j(t)) + C_D D_j(t)] dt, \quad j=0,1,2,3, \quad (17)$$

$$G_j = \begin{bmatrix} g_1 & 0 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 & 0 \\ 0 & 0 & g_3 & 0 & 0 \\ 0 & 0 & 0 & g_4 & 0 \\ 0 & 0 & 0 & 0 & g_5 \end{bmatrix} \quad \text{is weight matrix, } C_D \text{ is vector of "prices" per dose unit.}$$

The criterion (17) is formed by means of the model of soil state parameters (9):

$$\dot{V}_j = A_j V_j(t) + B_j D_j(t) + C_j F(t) - M_j X_j^*(t) - P_j S_j^*(t),$$

where $X_j^*(t)$, $S_j^*(t)$ are the optimal programs for the state parameters of crops and weeds obtained at the first stage.

The Hamiltonians for all control intervals are the same:

$$H_j = [(V_j^*(t) - V_j(t))^T G_j (V_j^*(t) - V_j(t)) + C_D D_j(t)] + \\ + \Psi_j^T [A_j V_j(t) + B_j D_j(t) + C_j F(t) - M_j X_j^*(t) - P_j S_j^*(t)], \quad (18)$$

where Ψ_j^T are vectors of linked variables for j -th control intervals.

The linked variable models are as follows:

$$\dot{\Psi}_j(t) = -\frac{\partial H_j(t)}{\partial V_j} = -[2G_j (V_j^*(t) - V_j(t)) + \tilde{A}_j^T \Psi_j(t)], \quad t \in (T_{2j}, T_{1j}), \quad \Psi_j(T_{2j}) = 0 \quad (19)$$

The algorithm for generating sequences of fertilizer application doses (optimal control programs) includes an iterative procedure for sequentially searching for successive approximations of fertilizer and irrigation dose vectors:

$$D_{i+1}^*(T_{1j}) = D_i^*(T_{1j}) - \Delta_i^* \frac{\partial H_j(T_{1j})}{\partial D_i(T_{1j})}, \quad (20)$$

$$D_{i+1}^*(T_{1j}) = D_i^*(T_{1j}) - \Delta_i^* (C_D + \tilde{B}_j^T \Psi_{i,j}(T_{1j})), \quad \text{if } D_{i+1}^*(T_{1j}) \in \Omega_j;$$

$$D_{i+1}^*(T_{1j}) = D_i^*(T_{1j}), \quad \text{if } D_{i+1}^*(T_{1j}) \notin \Omega_{T_{1j}}.$$

As a result of solving the task at the second stage, sequences of doses of fertilizer application and irrigation are formed over all management intervals $D_j^*(T_{1j})$, $j = 0,1,2,3$, which correspond to programs for changing the parameters of the soil environment $V_j^*(t)$, parameters sowing conditions $X_j^*(t)$ and weed biomass parameters $S_j^*(t)$.

The found optimal sequence of technological operations (program) does not yet take into account the direct effect of herbicides on the state of crop sowing in accordance with the models (2), (4). At the beginning of the procedure for the formation of a control program, such an effect cannot be taken into account, since the doses of treatments are not known a priori. Therefore, it is necessary to introduce one more external optimization cycle, in which such an influence is taken into account. To do this, it is necessary to close the entire procedure for the formation of an optimal control program for doses of herbicide treatment.

Models (4), (5) are used to solve the task:

$$\dot{X}_{ij} = A_{ij} X_{ij}(t) + B_{ij} V_j^*(t) + C_{ij} F(t) - D_{ij} G_{ij}(t),$$

$$\dot{S}_j = A_s S_j(t) + B_s V_j^*(t) - B_g G_j(t) + C_s F(t),$$

where $V_j^*(t)$ is the optimal program for changing the parameters of the soil environment, obtained at the second stage.

The goal of management at this stage is to select such doses of herbicide treatments at all management intervals that provide the best approximation of predictive programs for changing the parameters of the state of crops and parameters of weed biomass to the optimal programs found at the second stage.

This goal corresponds to the following optimality criteria for individual control intervals:

$$J_j = \int_{T_{1j}}^{T_{2j}} [(X_j^*(t) - X_j(t))^T G_{1j} (X_j^*(t) - X_j(t)) + (S_j^*(t) - S_j(t))^T G_{2j} (S_j^*(t) - S_j(t))] dt, \quad (21)$$

$$j = 0, 1, 2, 3$$

The algorithm for generating herbicide treatment programs is the following iterative procedure:

$$G_{j,i+1}^*(T_{1j}) = G_{j,i}^*(T_{1j}) - \Delta_i GR_{j,i}(T_{1j}),$$

$$GR_{j,i}(T_{1j}) = \frac{\partial H_{j,i}}{\partial G_{ji}}(T_{1j}) = D_j \Psi_{1j,i}(T_{1j}) + B_s \Psi_{2j,i}(T_{1j}). \quad (22)$$

At the first stage, the formation of fertilizer application programs was carried out without taking into account herbicide treatment programs. Given that fertilization and herbicide treatments are carried out simultaneously, fertilizer application programs need to be adjusted for herbicide application programs. For this, three global steps are introduced into the algorithm.

Step 1. The global cyclic variable $k = 1$ is accepted. The program of herbicide treatments $G^*(T_{1j})$ for all management intervals is substituted into the interval models, solving them from the beginning $j = 0$ to the end $j = 3$:

$$\dot{X}_{uj} = A_{uj} X_{uj}(t) + B_{uj} V_j^*(t) + C_{uj} F(t) - D_{uj} G_{uj}(t), \quad t \in (T_{1j}, T_{2j}), \quad (23)$$

$$\dot{S}_j = A_s S_j(t) + B_s V_j^*(T) - B_g G_j(t) + C_s F(T), \quad t \in (T_{1j}, T_{2j}), \quad (24)$$

in this case, the final solutions on the current interval are taken as initial ones for subsequent intervals: $X_0(T_{1j+1}) = X(T_{2j})$, $S_0(T_{1j+1}) = S(T_{2j})$. On the initial interval, the initial conditions common to the entire task are accepted: $X_0(T_{1j} = 0)$, $S_0(T_{1j} = 0)$. For the control interval, a global cyclic variable $k = 1$, $j = 3$ is taken, the criterion is calculated at the end of the growing season:

$$J_{k=1}(T_{13}) = [X_{uj=3}(T_{13}) - X_{uj=3}^*(T_{13})]^T G_u [X_{uj=3}(T_{13}) - X_{uj=3}^*(T_{13})] +$$

$$+ [S_{uj=3}(T_{13}) - S_{uj=3}^*(T_{13})]^T Q [S_{uj=3}(T_{13}) - S_{uj=3}^*(T_{13})]. \quad (25)$$

Step 2. If the criterion $J_{k=1}(T_{13})$ is less than the given value Δ , then STOP, otherwise the solutions of the models (4), (5) are transferred to point 1 of the first stage, and all stages of the task are repeated until obtaining a new criterion $J_k = 2(T_{13})$.

Step 3. If the criterion $J_k = 2(T_{13})$ is less than the criterion $J_k = 1(T_{13})$, then the solutions of the models (4), (5) must be transferred to point 1 of the first stage, otherwise STOP, and decisions are made for the previous criterion $J_k = 1(T_{13})$.

Results. The result of this work is the proposed algorithm for programmatic control of the state of agrocenosis with the sowing of spring wheat. This algorithm embodies a new theory of agrocenosis management and is implemented in a new specialized software product. The novelty and complexity of the algorithm requires

its approbation on experimental data, which should reflect the possibility of using the obtained results in practice.

At its core, approbation consists in establishing the fact of the stability of the control algorithm and its convergence to the minimum of the chosen optimality criterion. These conditions can be obtained only with the qualitative identification of all mathematical models used in the problem. It was carried out according to experimental data obtained for 2015–2021 at the Menkovsky branch of the Agrophysical Research Institute (Leningrad Province).

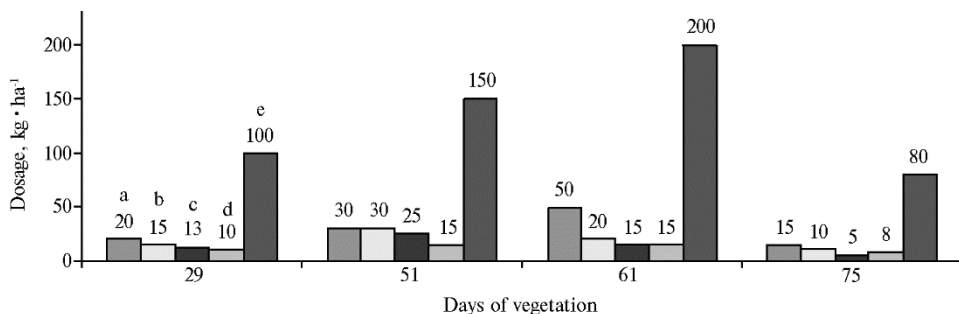


Fig. 1. The optimal program for the unified management of agrocnosis in terms of doses of mineral fertilizers and irrigation: a, b, c, d — doses of nitrogen, potassium, phosphorus and magnesium, respectively; e — irrigation rate, t · ha⁻¹.

The diagrams (Fig. 1, 2) show the results of optimization of agrocnosis management programs, including the sequence of doses of mineral fertilizers, irrigation and herbicide treatments. The program is focused on obtaining a given grain yield of 30 c/ha.

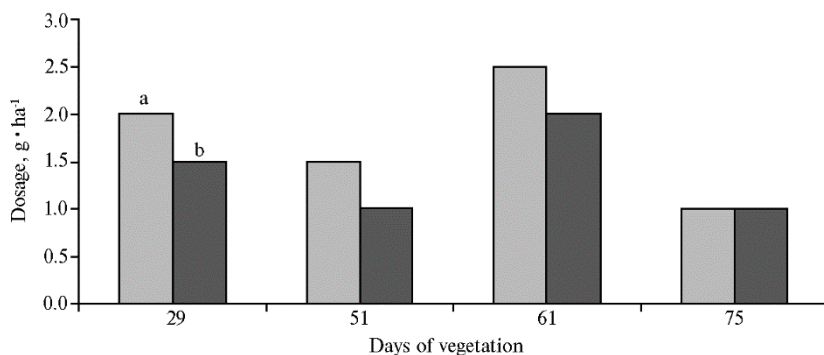


Fig. 2. Optimal program for unified management of agrocnosis by doses of herbicide treatments: a— herbicide 1 application rate, b — herbicide 2 application rate.

Here, technological operations were carried out at the onset of phenophases: $t = 29$ days (tillering), $t = 51$ days (heading), $t = 61$ days (flowering), $t = 75$ days (milky ripeness).

These programs correspond to the predicted dynamics of parameters of the biomass of spring wheat and two dominant weed species (Fig. 3, 4).

The given optimal control programs were obtained for 3 iterations of the global cycle of the algorithm, which corresponded to the following values of the optimality criterion [10] for the final sowing phenophase: iteration 1 — $J_u = 32$ (c/ha)², iteration 2 — $J_u = 14$ (c/ha)², iteration 3 — $J_u = 6$ (c/ha)². Note that all the optimality criteria used are quadratic functions, so the dimension of their productivity is taken in the square. The decrease of the optimality criterion over iterations of the global cycle of the algorithm proves its stability and successive approximation to the minimum of the optimality criterion.

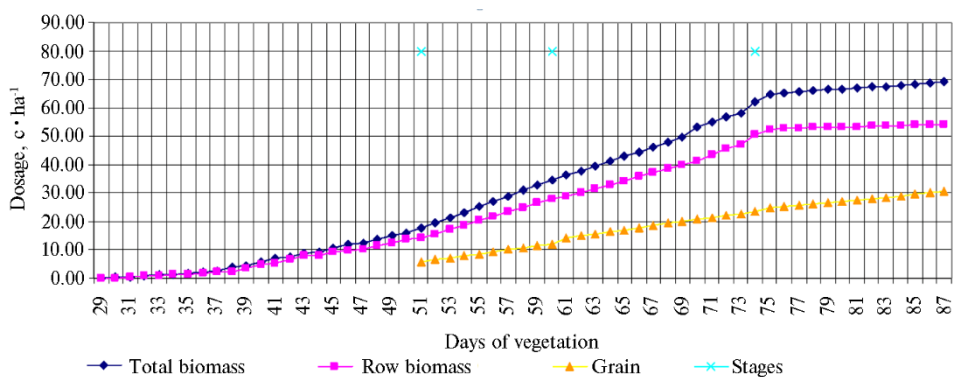


Fig. 3. Dynamics of biomass parameters of spring wheat sowing under the optimal program of unified management of agrocenosis.

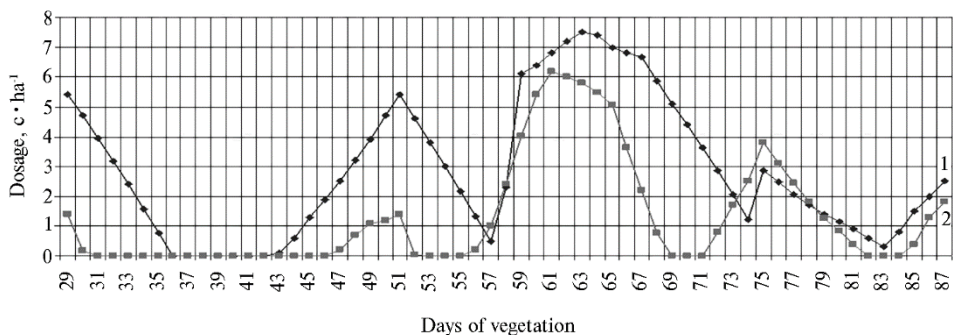


Fig. 4. Dynamics of biomass parameters of the dominant weed species under the optimal program of unified agrocenosis management: 1 — weed 1, 2 — weed 2.

As can be seen from the presented graph (see Fig. 3), the optimal program for the simultaneous application of fertilizers, watering and herbicide treatments ensures smooth growth and development of spring wheat sowing from germination to a given yield of 30 c/ha. The next graph (see Fig. 4) shows that weeds respond more dynamically to the technological operations of the optimal program, where stimulation with mineral fertilizers and irrigation causes an increase in weed biomass, which is suppressed by doses of herbicide treatments. At the same time, a general trend towards a decrease in weed biomass by the end of the growing season is manifested in the agrocenosis.

Based on the results of approbation of the problem, it can be argued that the proposed algorithm and specialized software package have characteristics sufficient to use this development as a means of intellectual support for an agronomist.

The algorithm and software package developed by us correspond to the concept of preventive management, which is considered as the main promising approach in measures to protect crops from weeds [20].

A number of models have been presented in the literature that quantify with some confidence the likelihood of outcomes for informed decision making, compare different management practices, and select options with the greatest long-term impact on the target group for agricultural units [20]. Selected examples provide an overview of models describing the distribution of weeds in crops, the use of such models in key areas of crop management, the quantitative findings and pragmatic results [20]. However, it is important to recognize that only few models (including ever developed decision support tools) have been widely applied to real pest management problems [38-41]. The main reasons cited are that practitioners consider the models to be inappropriate for local conditions and do not

have the time to study typical operating procedures; models do not take into account changes in the structure and number of weed populations, in the economics of crop production, and software standards are not sufficiently supported [50–53].

As can be seen from the analysis of available publications, all models known so far are designed for individual systems in agriculture and crop production (cultivated plants—weeds, cultivated plants—fertilizers, weeds—herbicides). We have set and solved a fundamentally new problem and developed a program for the unified management of agrocenosis (the system of cultivated plants—weeds—fertilizers—herbicides), which combines the listed particular tasks. The novelty of our invention is confirmed by the patent of the Russian Federation No. 2772889 “Method for the simultaneous differential application of liquid mineral fertilizers and herbicides and a device for its implementation” (dated May 26, 2022). The results of approbation showed the possibility of using this development in farm conditions.

Thus, we have proposed a new theory of programmed control of agrocenosis, focused on the implementation of the technological idea of the joint application of mineral fertilizers and herbicide treatments. It includes new mathematical models of parameters of the state of cultivated crops and weeds as part of agrocenosis, as well as new algorithms for the formation of optimal management programs. The algorithms are a four-stage sequential procedure which includes the creation of a program for the crop sowing development, the formation of a set of technological operations without herbicide treatments and a program with herbicide treatments carried out together with fertilization and irrigation, and, finally, the correction of programs for technological operations according to the herbicide treatments program. The proposed algorithm for optimal control programs avoids large dimensionality and complexity of the overall control task, ensures stability, a given crop yield and minimizes the weed biomass. The results obtained are a significant contribution to modern digitalization and intellectualization of agricultural technology management in precision farming.

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